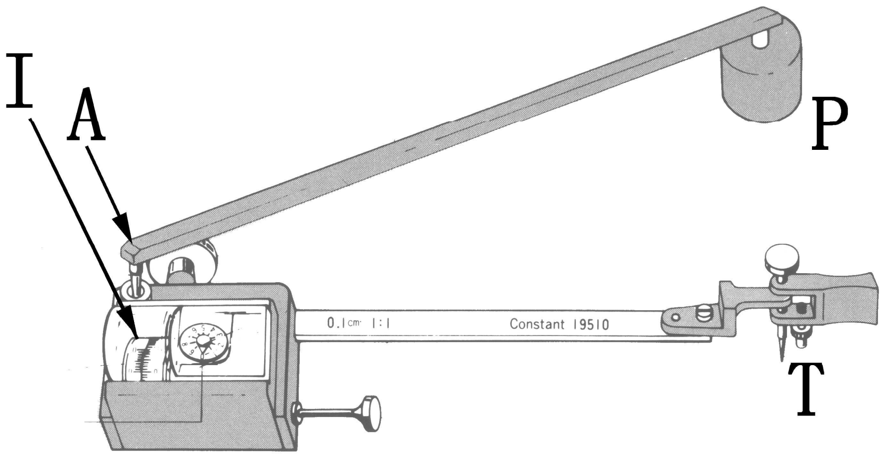
Planimeters: Basic Principles

1.Configuration

Planimeters are devices for measuring the area of complex curved-line figures. The device rotates with a fixed pole point (P) serving as its center. When the trace point (T) traces the periphery of a figure, the integrating wheel I rotates, and displays the area value measurement.



2.Regarding Forward Movements of the Integrating Wheel

Here, consideration is made of the case where measurements are made with a planimeter comprised, as in Fig.1, of:

- Polar Point: P
- Polar arm: PA
- Tracer arm: AT
- Integrating wheel: I
- Trace point: T.

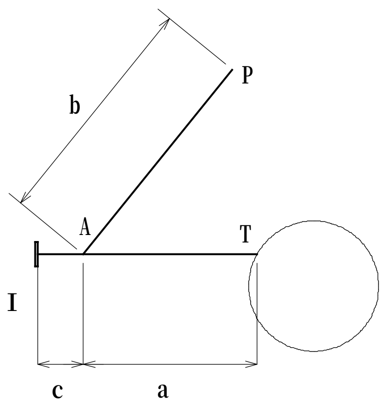


Fig.1

When the interval between points T to T" on the periphery have been traced, the planimeter moves as shown in Fig. 2a. This movement is broken down into the translation movement (*dn*) T to T', and the rotational motion ($d\phi$) from points T' to T".

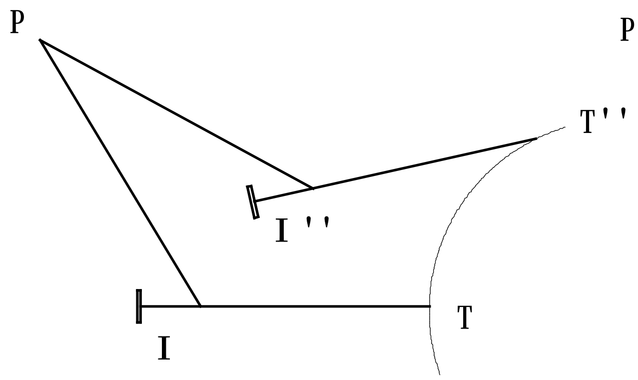


Fig.2a

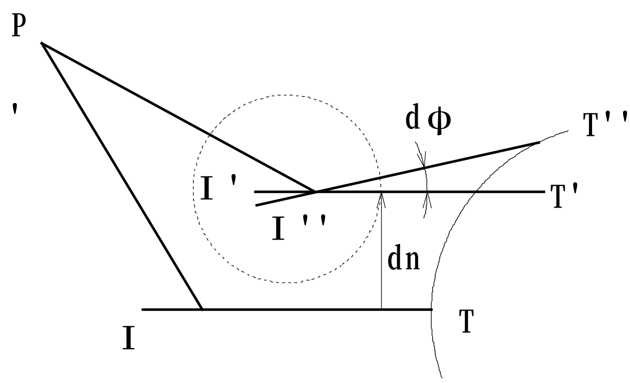


Fig.2b

The movement amount (*dl*) in the tangential direction of the integrating wheel I is:

$$dl = dn - c\,d\phi \qquad (1)$$

With this movement, the area swept by the tracer arm (AT) and the polar arm (PA) is, as depicted in Fig. 3:

$$dS = a\,dn + \pi a^2 \frac{d\phi}{2\pi} + \pi b^2 \frac{d\alpha}{2\pi}$$

$$= a\,dn + \frac{a^2 d\phi}{2} + \frac{b^2 d\alpha}{2} \qquad (2)$$

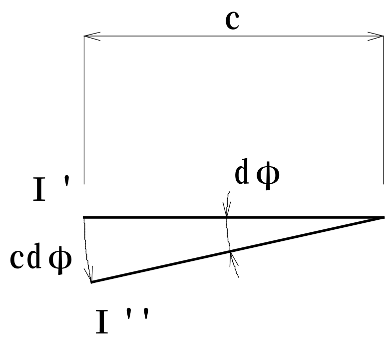


Fig.2b Expanded diagram

From (1) and (2):

$$dS = a(dl + c\,d\phi) + \frac{a^2 d\phi}{2} + \frac{b^2 d\alpha}{2} \qquad (3)$$

The area of the traced figure thus becomes:

$$S = \oint dS \qquad (4)$$

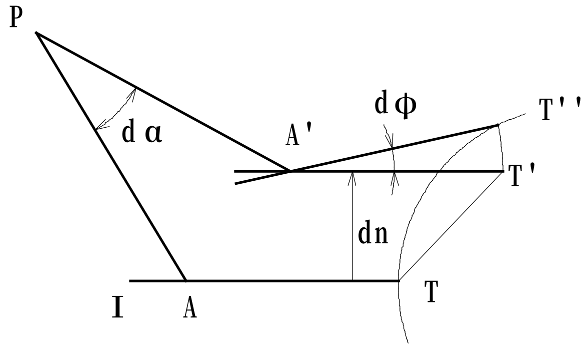


Fig.3

3.The Tracing of Figures

In the case where the area of a circle is sought, the area of the circle is calculated with the following formula.

$$S = \oint r\,d\phi$$

In the case of measurement with a planimeter, the area swept by the arm becomes:

$$S = \int_{T1}^{T3} r\,d\phi + \int_{T3}^{T1} r\,d\phi$$

$$= (S + S') - S' = S$$

Here, the area swept by the arm becomes equivalent to the area to be measured.

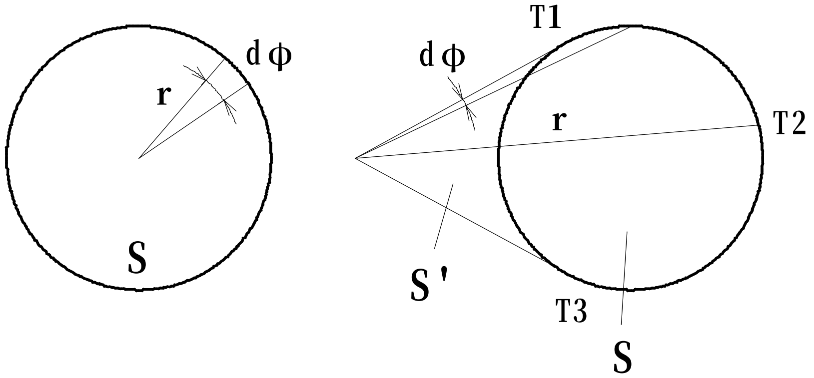


Fig.4

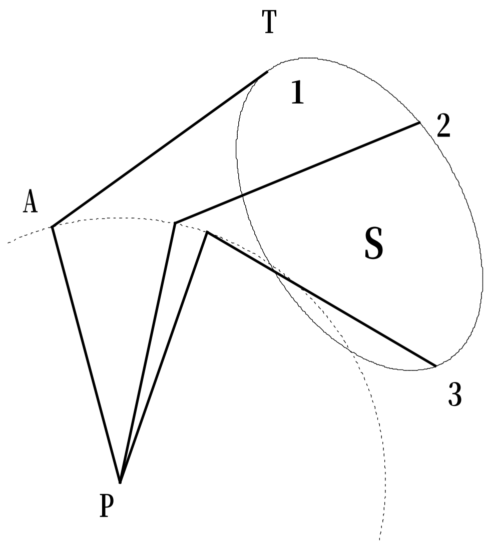


Fig.5

In the case of measurement of a figure of shape S as in Fig.5, the polar arm (PA) makes a single, left-to-right return, while the tracer arm makes a single, up-down return. At that time, the angle of rotation when the polar arm is moved from the leftmost point to the rightmost point becomes:

$$\int_1^3 d\alpha = \alpha_1$$

When returned from the rightmost point to the leftmost point, this is:

$$\int_3^1 d\alpha = -\alpha_1$$

Thus, the total angle of rotation is:

$$\oint d\alpha = 0$$

Similarly,

$$\oint d\phi = 0$$

Therefore, Formula (4) is:

$$S = \oint dS = \oint adl + (ac + \frac{a^2}{2})\oint d\phi + \frac{b^2}{2}\oint d\alpha = \oint adl = al$$

a: length of the tracer arm
l: Movement advancement amount of integrating wheel in the tangential direction

If r is the radius of the integrating wheel, and n is the number of integrating wheel rotations, then:

$$l = 2\pi rn$$

Thus:

$$\boxed{S = 2\pi rna}$$

The area sought is thus proportional to the rotation amount of the integrating wheel.